We can change the hypothesis and conclusion to form 3 related conditional statements.

- **CONVERSE**
- **INVERSE**
- **CONTRAPOSITIVE**
Ex.

Given that the original conditional statement is true, write the related conditional statements.

Original: If you are an ape, then you like bananas.

Converse: If you like bananas, then you are an ape.

Original: If you are an ape, then you like bananas.

Inverse: If you are not an ape, then you don't like bananas.

Original: If you are an ape, then you like bananas.

Contrapositive:
   If you don't like bananas, then you are not an ape.
Practice: Write the **converse**, **inverse**, and **contrapositive** of each conditional statement. Then determine (using your own knowledge of the world) whether each is **true** or **false**.

1. If you live in Salem, NH, then you live in the USA.
   - **converse:** IF **YOU LIVE IN USA**, THEN **YOU LIVE IN SALEM**. \( \text{F} \)
   - **inverse:** IF **YOU DO NOT LIVE IN SALEM**, THEN **YOU DO NOT LIVE IN USA**. \( \text{F} \)
   - **contrapositive:** IF **YOU DO NOT LIVE IN USA**, THEN **YOU DO NOT LIVE IN SALEM NH**. \( \text{F} \)

2. An acute angle is an angle whose measure is less than \(90^\circ\).
   - **converse:** IF **AN ANGLE IS ACUTE**, THEN **ITS MEASURE IS \(< 90^\circ\)**. \( \text{T} \)
   - **inverse:** IF **AN ANGLE MEASURES \(< 90^\circ\)**, THEN **IT IS ACUTE**. \( \text{T} \)
   - **contrapositive:** IF **AN ANGLE IS NOT ACUTE**, THEN **ITS MEASURE IS \(\geq 90^\circ\)**. \( \text{T} \)

3. All rectangles are quadrilaterals.
   - **converse:**
   - **inverse:**
   - **contrapositive:**
The converse and inverse always have the same truth value.

The original and contrapositive always have the same truth value.
If the original, converse, inverse, and contrapositive are all true, we call the original a bijective statement.

A bijective statement is a definition.
biconditional statement:
a biconditional statement is a way to write TWO conditional statements using only one statement.

**a biconditional will (usually) have the phrase "if and only if" between the hypothesis and conclusion

Ex.
An angle is acute if and only if its measure is less than 90°.

This biconditional statement says...
*If an angle is acute, then its measure is less than 90°.
AND
*If the measure of an angle is less than 90°, then it is acute.

A true biconditional statement is one in which both statements are true.
*The ___________ and ___________ always have the same truth value.

*The ___________ and ___________ always have the same truth value.

**If all four related conditional statements are TRUE, we can write a ________________.
*The converse and inverse always have the same truth value.

*The original and contrapositive always have the same truth value.

**If all four related conditional statements are TRUE, we can write a **biconditional statement**.

A definition is a biconditional statement.
2.4 Deductive Reasoning

Now we can start making conjectures using our two laws...

Law of Detachment and Law of Syllogism
Law of Detachment:

Given a conditional statement,

\[ \text{IF } (\text{HYPOTHESIS}), \text{THEN } (\text{CONCLUSION}). \]

And a statement that matches the hypothesis,

\[ (\text{HYPOTHESIS}) \]

You can conclude a statement that matches the conclusion.

\[ (\text{CONCLUSION}) \]

Ex. If a polygon has 4 right angles, then it is a rectangle. ABCD is a polygon with 4 right angles. So, it must follow that...

\[ \text{ABCD IS A RECTANGLE}. \]
Law of Syllogism:

Given two conditional statements that have a shared hypothesis/conclusion,

\[ \text{IF (HYPOTHESIS 1), THEN (CONCLUSION 1).} \]
\[ \text{IF (HYPOTHESIS 2, SAME), THEN (CONCLUSION 2).} \]

You can conclude another conditional statement.

\[ \text{IF (HYPOTHESIS 1), THEN (CONCLUSION 2).} \]

Ex. If you live in Salem, NH, then you live in New England.
If you live in New England, then you live in the US.
So, it must follow that...

\[ \text{IF you live in Salem, NH, then you live in the US.} \]
Now we can start making conjectures using our two laws...

**Law of Detachment** and **Law of Syllogism**

Ex. Make a conjecture based on the following conditional statements.

If you are an ape, then you love bananas.
Karen is an ape.

Karen loves bananas.
Ex. Make a conjecture based on the following conditional statements.

If you are an ape, then you love bananas.
If you love bananas, then you like wearing hats.
Ex. Make a conjecture if possible. If not possible, write no valid conclusion.

(1) If two angles are complementary to the same angle, then the angles are congruent.

(2) $\angle E$ and $\angle F$ are complimentary to $\angle G$. 
Ex. Make a conjecture if possible. If not possible, write *no valid conclusion*.

(1) If a dog eats Superdog Dog Food, he will be happy.

(2) Rover (the dog) is happy.