Geometry
Properties of Parallelograms

**Definition:**
A parallelogram is a quadrilateral with two sets of parallel sides.

**Property:** Consecutive angles of a parallelogram are supplementary.

**Theorem:** Opposite angles of a parallelogram are congruent.

**Theorem:** Opposite sides of a parallelogram are congruent.

**Theorem:** Diagonals of a parallelogram bisect each other.

**Example #1:** Quadrilateral $ABCD$ is a parallelogram. Find $x$ given that $AD = 7x + 5$, $AB = 5x + 9$, and $BC = 14x - 2$.

\[
\begin{align*}
AD &= BC \\
7x + 5 &= 14x - 2 \\
7 &= 7x \\
1 &= x
\end{align*}
\]

**Definition:**
A rectangle is a parallelogram with four right angles.

**Theorem:** Diagonals of a rectangle are congruent.

**Example #2:** Rectangle $ABCD$ has diagonals that intersect at $E$. $AC = (6y - 24)$ cm and $BE = (2y + 10)$ cm. Find the length of each diagonal.

\[
\begin{align*}
AC &= BD \\
AC &= BE + ED \\
AC &= BE + BE \\
6y - 24 &= 2y + 10 + 2y + 10 \\
6y - 24 &= 4y + 20 \\
2y &= 44 \\
y &= 22
\end{align*}
\]

\[
AC = 108 \text{ cm} \\
BD = 108 \text{ cm}
\]
Definition:
A rhombus is a parallelogram with four congruent sides.

Theorem: Diagonals of a rhombus are perpendicular.

Theorem: Diagonals of a rhombus bisect opposite angles.

Example #3: \(ABCD\) is a rhombus.
\[m\angle DEC = 4x + 10\] and \[m\angle ABC = 3x + 4.\]
Find \(m\angle CAB\).

\[\begin{align*}
m\angle DEC &= 90 \\
4x + 10 &= 90 \\
4x &= 80 \\
x &= 20
\end{align*}\]

\[\begin{align*}
m\angle ABC &= 3x + 4 \\
&= 3(20) + 4 \\
&= 64^\circ
\end{align*}\]

\[m\angle DAB = \frac{64}{2} = 32^\circ\]

In \(\triangle AEB\)...
\[90 + 32 + m\angle CAB = 180\]
\[m\angle CAB = 58^\circ\]

Definition:
A square is a parallelogram with four congruent sides and angles.

Example #4: \(JKLM\) is a square. Find the values of \(x\) and \(y\).

\[\begin{align*}
7y - 1 &= 90 \\
7y &= 91 \\
y &= 13
\end{align*}\]

\[\begin{align*}
JK &= 3y + 2 \\
&= 3(13) + 2 \\
&= 39 + 2 \\
&= 41
\end{align*}\]

\[JK = KL\]
\[41 = 5x + 1\]
\[40 = 5x\]
\[8 = x\]