4.4/4.5 Counting Rules

Fundamental Counting Rule:

In a sequence of n events in which the
1st has $k_1$ possibilities
2nd has $k_2$ possibilities,
etc.

Total # of possibilities is $k_1 \cdot k_2 \cdot \ldots \cdot k_n$
Ex. You flip a coin, roll a die, pick a card

Total # of possibilities = \( \frac{2 \cdot 6 \cdot 52}{624} \)
Ex. At a sandwich shop, you choose a bread (and whether you want it toasted), a cheese, a meat, and a spread.

There are 5 bread choices, 3 cheeses, 4 meats, and 6 spreads.

How many possibilities? $5 \cdot 3 \cdot 4 \cdot 6 = 720$
Ex. How many blood type labels are there?

Blood Types: A, B, AB, O

Blood can be: Rh+ or Rh-

Donor can be Male or Female  \( 4 \times 2 \times 2 \)
Ex. A NH license plate needs 7 characters (numbers and/or letters). How many different license plates are possible?

\[
\underbrace{36 \cdot 36 \cdot 36 \cdot 36 \cdot 36 \cdot 36} = 36^7
\]

7.84e10 = 7.84 \times 10^{10} = \text{78,400,000,000}
What if you are not allowed to repeat a character?

\[ \frac{36 \cdot 35 \cdot 34 \cdot 33 \cdot 32 \cdot 31 \cdot 30}{4 \cdot 2 \times 10^{10} = 4 200 000 000 000} \]
22 people and each gets a unique prize

1st → $1
2nd → $2
3rd → $3

How many different outcomes?

22, 21, 20, 19, 18, 17, 16, 15, 14, 13, 12, 11, 10, 9, 8, 7, 6, 5, 4, 3, 2, 1

22nd → $22

\[ 1 \times 10^{21} = 22! \]
Counting Rules

\[ k! = \text{"k factorial"} = k \cdot (k-1) \cdot (k-2) \cdot \ldots \cdot 2 \cdot 1 \]

\[ 0! = 1 \]

Ex. \[ 8! = 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 40320 \]
Ex. There are 22 people in the class. Ms. Tecce will give out 3 different prizes to 3 different people. How many different outcomes?

\[
\frac{22 \cdot 21 \cdot 20}{\$1\text{ million} \times \text{At in Stats} \times \text{lollipop}} = 9240
\]

\[
\frac{22!}{19! \cdot (22-3)!} = 22!
\]

*The results Hunter, Colin, Osiris and Osiris, Colin, Hunter are different!*

So… ORDER MATTERS
Permutation: an arrangement of \( n \) objects in a specific order (only use \( r \) of the objects)

\[
\begin{align*}
\text{without replacement} \\
nPr &= \frac{n!}{(n-r)!} \\
nPr &= \text{out of \( n \), Pick \( r \) (order matters)}
\end{align*}
\]

Ex. \( 22P_3 = \text{Pick 3 out of 22} \)

\[
\begin{align*}
&= \frac{22!}{(22-3)!} = \frac{22!}{19!} = 22 \cdot 21 \cdot 20 \\
&= 9240
\end{align*}
\]
Ex. Ms. Tecce picks 3 out of 22 to get a lollipop.
How many different outcomes?

The results Celia, Lacey, Jill are NOT different.
Jill, Lacey, Celia

So... ORDER DOES NOT MATTER
Combination: an arrangement of n objects NOT in a specific order (only use r objects)

\[
\binom{n}{r} = \frac{n!}{(n-r)! \cdot r!}
\]

\[
\binom{n}{r} = \text{out of } n, \text{ Choose } r \quad (\text{order does NOT matter})
\]

without replacement