Name the term that best describes the notation.

1. \( D \)  Point of tangency
2. \( \overline{FH} \)  tangent
3. \( \overline{CD} \)  radius
4. \( \overline{AB} \)  chord
5. \( C \)  center
6. \( \overline{AD} \)  diameter
7. \( \overline{AB} \)  secant
8. \( \overline{DE} \)  tangent
\[ x = \]

Triangle \(\triangle ABC\) and \(\triangle ACD\) share a common tangent at point \(A\). The tangents from a single point are equal. Therefore:

- \(AB = AD\)
- \(7x + 4 = 3x + 68\)

Solving for \(x\):

\[ 4x = 64 \]
\[ x = 16 \]
\[ \widehat{BC} = 50, \widehat{AB} \cong \widehat{ED}, \widehat{AE} = \underline{\underline{80^\circ}}} \]

- Diameter is ⊥ bisector of chord

- \[ m \widehat{AB} + m \widehat{AE} + m \widehat{ED} = 180 \]
- \[ 50 + m \widehat{AE} + 50 = 180 \]
- \[ m \widehat{AE} = 80^\circ \]
In the diagram, $C$ is the center of both circles, and the radii of the circles are 8 and 17. If $JL$ is tangent to the circle of radius 8, find the length of $JL$.

\[ x^2 + 8^2 = 17^2 \]
\[ x^2 + 64 = 289 \]
\[ x^2 = 225 \]
\[ x = 15 \]

\[ JL = x + x \]
\[ JL = 30 \]
Opposite angles are supplementary in an inscribed quadrilateral.

\[2y - 3 + y + 5 = 180\]

\[3y + 2 = 180\]

\[3y = 178\]

\[y = \frac{178}{3}\]

\[3x + 90 = 180\]

\[3x = 90\]

\[x = 30\]
Find the diameter of a circle if a 48 cm chord is 7 cm from the center.

\[ 7^2 + 24^2 = r^2 \]
\[ 49 + 576 = r^2 \]
\[ 625 = r^2 \]
\[ 25 = r \]

\[ 50 = d \]
\[ x = \] 

\[
\text{angle} = \frac{A\text{RC} + \text{arc}}{2}
\]

\[ 115 = \frac{105 + x}{2} \]

\[ 230 = 105 + x \]

\[ 125 = x \]
\[ x = \text{_______} \]

\[ \text{angle} = \frac{\text{ARC}_1 - \text{ARC}_2}{2} \]

\[ 38 = \frac{180 - x}{2} \]

\[ 76 = 180 - x \]

\[ -104 = -x \]

\[ 104 = x \]
$x = \underline{\ ? }$

\[ \text{angle} = \frac{\text{ARC} - \text{arc}}{2} \]

\[ 17 = \frac{3x + 1 - 42}{2} \]

\[ 34 = 3x - 41 \]

\[ 75 = 3x \]

\[ 25 = x \]
\[ x = \phantom{0} \]

\[
\text{angle} = \frac{\text{arc}}{2} \\
76 = \frac{x}{2} \\
152 = x
\]
\[ \text{angle} = \frac{\text{arc} - \text{arc}}{2} \]

\[ \chi = \frac{252 - 108}{2} \]

\[ \chi = \frac{144}{2} \]

\[ \chi = 72^\circ \]
Tell whether $\overline{AB}$ is tangent to the circle.

$AC^2 + AB^2 = BC^2$

$18^2 + 80^2 = 82^2$

$324 + 6400 = 6724$

$6724 = 6724$

Yes! $\angle A$ is right, so $\overline{AB}$ is tangent to the circle.
Given \( \odot O \) with \( m \angle OCB = 75 \).

Find the measure of \( \angle A \).

\[ \text{Let } m \angle A = x \]

\[ m \angle AC + m \angle CB = 180^\circ \]

\[ 180 - 2x \]

\[ 2x^\circ \text{ Inscribed angle is } \angle A \]

So arc measure is 2 \( \angle \) angle

\[ \text{arc} = 2 \cdot \text{angle} \]

\[ AO = OC \text{ since both are radii} \]

so \( \triangle AOC \) is isosceles with base angles \( \angle ACO \) and \( \angle OAC \)

\[ 180 - (x) - (180 - 2x) \]

\[ 180 - x - 180 + 2x \]

\[ \angle ACB \Rightarrow x + (x + 75) + 90 - x = 180 \]

\[ x + 165 = 180 \]

\[ x = 15^\circ \]

\[ m \angle A = 15^\circ \]
Given $\odot O$ with $m\widehat{SM} = 80$, $m\widehat{PS} = 90$, and $m\widehat{PT} = 70$. Find $m\widehat{TM}$, $m\angle P$, $m\angle M$, and $m\angle T$.

$m\widehat{TM} = 120^\circ$

$m\angle P = 100^\circ$

$m\angle M = 80^\circ$

$m\angle T = 85^\circ$
If \( AB = 20 \) and \( PQ = 2 \), find the diameter of the circle.

\[
(x - 2)^2 + 10^2 = x^2
\]
\[
(x - 2)(x - 2) + 100 = x^2
\]
\[
x(x - 2) = 2(x - 2) + 100 = x^2
\]
\[
x^2 - 2x - 2x + 4 + 100 = x^2
\]
\[
x^2 - 4x + 104 = x^2
\]
\[
-4x + 104 = 0
\]
\[
104 = 4x
\]
\[
26 = x
\]
\[
52 = d
\]
Given $\odot O$ with radius 10, $\overline{AO} \perp \overline{AB}$ and $m\overarc{AC} = 60$. Find $BC$. 

30°-60°-90° triangle... so $OB = 2\cdot AO$

$BC = 10$
Given two concentric circles with radii of lengths 16 and 20. Find the length of a chord of the larger circle that is tangent to the smaller circle.

\[ x^2 + 16^2 = 20^2 \]
\[ x^2 + 256 = 400 \]
\[ x^2 = 144 \]
\[ x = 12 \]

\[ \text{length of chord} = 2x \]
Given \( \odot O \) with tangents \( \overline{DA} \) and \( \overline{DC} \), \( m \overline{AC} = 120 \), 
\( m \overline{AE} = 84 \), and \( m \overline{EG} = 58 \).

Find the measure of the numbered angles.
Write the equation of a circle that has endpoints of a diameter at (-3, 8) and (5, -2).

\[ d = \sqrt{(-3-5)^2 + (8-(-2))^2} \]
\[ = \sqrt{(-8)^2 + (10)^2} \]
\[ = \sqrt{64 + 100} \]
\[ d = \sqrt{164} \]
\[ r = \frac{\sqrt{164}}{2} \]

\[ (x-h)^2 + (y-k)^2 = r^2 \]
\[ (x-1)^2 + (y-3)^2 = \left(\frac{\sqrt{164}}{2}\right)^2 \]
\[ (x-1)^2 + (y-3)^2 = 41 \]
Find the diagonal of a rectangle whose sides are 20 dm and 48 dm.

\[ a^2 + b^2 = c^2 \]
\[ 20^2 + 48^2 = x^2 \]
\[ 400 + 2304 = x^2 \]
\[ 2704 = x^2 \]
\[ \sqrt{2704} = x \]
\[ 52 = x \]
Find the perimeter of an isosceles triangle whose base is 16 and whose height is 15.

\[
\begin{align*}
17^2 + 15^2 &= c^2 \\
8^2 + 15^2 &= x^2 \\
64 + 225 &= x^2 \\
289 &= x^2 \\
17 &= x
\end{align*}
\]

Perimeter = 17 + 17 + 16

= 50
A man travels 1 km north, 2 km east, 3 km north, and 4 km east. How far is he from his starting point?

$$a^2 + b^2 = c^2$$

$$(\text{run})^2 + (\text{rise})^2 = c^2$$

$$b^2 + 4^2 = c^2$$

$$3b + 16 = c^2$$

$$\sqrt{52} = c$$
In the rectangular solid, every two intersecting edges are perpendicular. If $AE = 3$, $AB = 4$, and $BC = 12$, find the length of the diagonals $BE$ and $BH$.

$\triangle AEB: BE \Rightarrow 3^2 + 4^2 = x^2$

$9 + 16 = x^2$

$25 = x^2$

$x = 5$  \( \text{BE} = 5 \)

$\triangle CDB: 4^2 + 12^2 = y^2$

$16 + 144 = y^2$

$160 = y^2$

$\sqrt{160} = y$

$BD = \sqrt{160}$

$\triangle HDB: z^2 + (\sqrt{160})^2 = z^2$

$9 + 160 = z^2$

$169 = z^2$

$z = 13$  \( BH = 13 \)
In this rectangular solid, $\overline{AG}$ and $\overline{EC}$ are diagonals. If $AB = 9$, $BF = 12$, and $AD = 8$, find $AG$ and $EC$. 

\[ \triangle AGB: \]
\[ 9^2 + 12^2 = (AF)^2 \]
\[ 81 + 144 = (AF)^2 \]
\[ 225 = (AF)^2 \]
\[ 15 = AF \]  

\[ \triangle AFG: \]
\[ 15^2 + 8^2 = (AG)^2 \]
\[ 225 + 64 = (AG)^2 \]
\[ 289 = (AG)^2 \]
\[ 17 = AG \]  

\[ \triangle EAB: \]
\[ 12^2 + 9^2 = (EB)^2 \]
\[ 144 + 81 = (EB)^2 \]
\[ 225 = (EB)^2 \]
\[ 15 = EB \]  

\[ \triangle ECB: \]
\[ 8^2 + 15^2 = (EC)^2 \]
\[ 64 + 225 = (EC)^2 \]
\[ 289 = (EC)^2 \]
\[ 17 = EC \]
\[
\begin{align*}
&x^2 + 1^2 = \sqrt{3}^2 \\
&4 + 1 = x^2 \\
&5 = x^2 \\
&\sqrt{5} = x \\
&\sqrt{3} + 1 = c^2 \\
&4 = c^2 \\
&2 = c \\
&\sqrt{3} + 1 = b^2 \\
&2 = b^2 \\
&3 = b \\
&\sqrt{3} = b \\
\end{align*}
\]
Pythagorean Triplet
8, 15, 17

17

8

6

15

25

26

10

24

Triplet 7, 24, 25

8, 6, 10 (3, 4, 5)

Triplet 10, 24, 26 (5, 12, 13)

HG Unit 5 Review Packet 2018
Woody Woodpecker pecked at a 17-m wooden pole until it cracked and the upper part fell, with the top hitting the ground 10 m from the foot of the pole. Since the upper part had not completely broken off, Woody pecked away where the pole had cracked. How far was Woody above the ground?

\[ a^2 + b^2 = c^2 \]
\[ x^2 + 10^2 = (17 - x)^2 \]
\[ x^2 + 100 = 289 - 34x + x^2 \]
\[ 100 = 289 - 34x \]
\[ 34x = 189 \]
\[ x = \frac{189}{34} = 5 \frac{19}{34} \text{ m} \]
**RHOM** is a rhombus with diagonals *RO* = 48 and *HM* = 14. Find the perimeter of the rhombus.

[Hint: Recall that the diagonals of a rhombus are \( \perp \) bisectors of each other.]

\[
\begin{align*}
7^2 + 24^2 &= x^2 \\
49 + 576 &= x^2 \\
625 &= x^2 \\
25 &= x \\
\text{Perimeter} &= RH + HO + MO + RM \\
&= 25 + 25 + 25 + 25 \\
&= 100
\end{align*}
\]
The lengths of the diagonals of a rhombus are in the ratio 2:1. If the perimeter of the rhombus is 20, find the sum of the lengths of the diagonals.

[Hint: Recall that the diagonals of a rhombus are \( \perp \) bisectors of each other.]

\[
\begin{align*}
\text{perimeter} &= 20 \quad \text{and all 4 sides are } \approx \\
ax^2 + bx^2 &= c^2 \\
(1x)^2 + (2x)^2 &= 5^2 \\
5x^2 &= 25 \\
x^2 &= 5 \\
x &= \sqrt{5}
\end{align*}
\]

\[
\text{sum of lengths of diagonals} = \text{red + purple} \\
2\sqrt{5} + 2\sqrt{5} + 1\sqrt{5} + 1\sqrt{5} = 3\sqrt{5}
\]
In $\triangle ABC$, $\angle C$ is a right angle, $AC = 20$, and $BC = 15$. Find the length of $AB$ and the length of the altitude to the hypotenuse.

$\triangle CDB: \ x^2 + y^2 = 15^2 \Rightarrow x^2 + y^2 = 225$

$\triangle CDA: \ x^2 + w^2 = 20^2 \Rightarrow x^2 + w^2 = 400$

$\triangle ABC: \ 15^2 + 20^2 = (w+y)^2 \Rightarrow 225 + 400 = (w+y)^2$

$625 = (w+y)^2$

$25 = w + y$

$25 - w = y$

$(25-w)^2 = w^2 - 175$

$625 - 50w + w^2 = w^2 - 175$

$625 - 50w = -175$

$800 = 50w$

$16 = w$

$y = 25 - 16$

$y = 9$

$x^2 + (4)^2 = 225$

$x^2 = 144$

$x = 12$
\[ a^2 + b^2 = c^2 \]
\[ 16^2 + 10^2 = x^2 \]
\[ 256 + 100 = x^2 \]
\[ 356 = x^2 \]
\[ \sqrt{356} = x \]
\[ a^2 + b^2 = c^2 \]
\[ 16^2 + x^2 = 20^2 \]
\[ 256 + x^2 = 400 \]
\[ x^2 = 144 \]
\[ x = 12 \]
\[ a^2 + b^2 = c^2 \]
\[ x^2 + 24^2 = 26^2 \]
\[ x^2 + 576 = 676 \]
\[ x^2 = 100 \]
\[ x = 10 \]
In $\triangle ABC$, $\angle A$ is a right angle and $m\angle B = m\angle C = 45$. Given that $BC = 6$, find $AB$.

\[
\begin{align*}
a^2 + b^2 &= c^2 \\
x^2 + x^2 &= 6^2 \\
2x^2 &= 36 \\
x^2 &= 18 \\
x &= \sqrt{18} \\
AB &= \sqrt{18}
\end{align*}
\]
A triangle has side lengths 5, 14, and 18. Is the triangle acute, obtuse, or right (show all work).

\[ a \quad b \quad c \]

\[ a^2 + b^2 \quad ? \quad c^2 \]
\[ 5^2 + 14^2 \quad ? \quad 18^2 \]
\[ 25 + 196 \quad ? \quad 324 \]
\[ 221 \quad < \quad 324 \]

Since \( a^2 + b^2 < c^2 \), the triangle is obtuse.
Find the measure of the arc or central angle indicated. Assume that lines which appear to be diameters are actual diameters.

\[ m\widehat{WV} \]

\[ 130° + 65° + 65° + m\widehat{VW} + m\widehat{VW} = 360° \]

Now plug in!

\[ m\widehat{VW} = 50° \]

\[ 12x - 2 = 5x + 10 + 5x + 10 \]

\[ 12x - 2 = 10x + 20 \]

\[ 2x = 22 \]

\[ x = 11 \]
Find the measure of the arc or central angle indicated. Assume that lines which appear to be diameters are actual diameters.

\[ m \angle VST \]

\[ m \widehat{TU} + m \widehat{UV} + m \widehat{TV} = 360 \]
\[ (-27x-3) + (-27x-3) + (-37x+2) = 360 \]
\[ -91x - 4 = 360 \]
\[ -91x = 364 \]
\[ x = -\frac{364}{91} = -\frac{364}{91} \]

\[ m \angle VST = -27x - 3 \]
\[ = -27\left(-\frac{364}{91}\right) - 3 \]
\[ = 113\frac{82}{91}^\circ \]
Find the length of the segment indicated. Round your answer to the nearest tenth if necessary.

So...

\[ x + 7 = 15.9 \]

\[ x = 8.9 \]
Find the area and circumference of the circle.

\[ A = \pi r^2 \]

\[ A = \pi (9)^2 \]

\[ A = 81\pi \text{ m}^2 \]

\[ C = 2\pi r \]

\[ C = 2\pi (9) \]

\[ C = 18\pi \text{ m} \]

MUST include units!!
Find the radius of each circle. Use your calculator's value of π. Round your answer to the nearest tenth.

\[ \text{circumference} = 69.1 \text{ yd} \]

\[ C = 2\pi r \]

\[ \frac{69.1}{2\pi} = r \]

*If you got 108.5, be sure to put \( (\text{ around } 2\pi \) when typing in your calculator!*

\[ 11.0 \text{ yd} \approx r \]
Find the diameter of each circle. Use your calculator's value of π. Round your answer to the nearest tenth.

area = 201.1 in²

\[ A = \pi r^2 \]

\[ \frac{201.1}{\pi} = r^2 \]

\[ \sqrt{\frac{201.1}{\pi}} = r \]

\[ 8.0 \text{ in.} \approx r \]
Find the radius of a circle so that its area and circumference have the same value.

\[ A = \pi r^2 \quad C = 2\pi r \]

\[ \frac{\pi r^2}{\pi} = \frac{2\pi r}{\pi} \]

\[ \frac{r^2}{r} = \frac{2r}{r} \]

\[ r = 2 \]
Find the measure of the arc or angle indicated.

\[ 2 \chi = 80 \]
\[ \chi = 40^\circ \]
Find $m\angle NLM$

\[13x - 10 + 7x - 10 = 180\]
\[20x = 200\]
\[x = 10\]

\[m\angle NLM = \frac{60^\circ}{2} = 30^\circ\]
Determine if line $AB$ is tangent to the circle.

Is $\angle A$ right?

Is the $\triangle$ right?

$a^2 + b^2 = c^2$

$12^2 + 16^2 = (8+12)^2$

$144 + 256 = 400$

$400 = 400 \checkmark$

Yes, since $\angle A$ is right, $\overrightarrow{AB}$ is tangent to the circle.
Find the perimeter of each polygon. Assume that lines which appear to be tangent are tangent.

Perimeter = 23 + 22.7 + (14 + 13.7)
= 73.4
117° + 90° + 90° + x = 360°

x = 63°
\[ \text{angle} = \frac{\text{ARC} + \text{arc}}{2} \]

\[ 2 \cdot 85 = \frac{70 + x}{2} \cdot 2 \]

\[ 170 = 70 + x \]

\[ 100^\circ = x \]
2\cdot \text{angle} = \text{arc}

2(88) = 15x + 11

176 = 15x + 11

165 = 15x

11 = x
angle = \frac{\text{ARC} + \text{arc}}{2}

2 \cdot 124 = \frac{197 + 9x - 3}{2} \cdot 2

248 = 197 + 9x - 3

248 = 194 + 9x

\frac{54}{9} = x

\frac{54}{3} = x
$2 \cdot \angle Q = \text{arc}$

$2x = 146$

$x = 73^\circ$
\[ \text{angle} = \frac{\text{ARC} - \text{arc}}{2} \]

\[ 2 \cdot 44 = \frac{x - 65}{2} \]

\[ 88 = x - 65 \]

\[ 153 = x \]
Find the center and radius. Then graph the circle.

\((x-h)^2 + (y-k)^2 = r^2\)  \(\rightarrow\)  
\(x^2 + (y-3)^2 = 14\)

\(r^2 = 14\)  \(\rightarrow\)  
\(r = \sqrt{14}\)

- **center:** \((0, 3)\)
- **radius:** \(\sqrt{14} \approx 3.7\)
ABCDEF has vertices at \(A(8, 9), B(-3.4, 9), C(-8, 2), D(-8, -4), E(-1.2, -4),\) and \(F(5, 0).\)

Find the perimeter of \(ABCDEF\) to the nearest hundredth.

\[
AB = \sqrt{(8-(-3.4))^2 + (9-9)^2} = 11.4
\]
\[
BC = \sqrt{(-3.4-8)^2 + (9-2)^2} = \sqrt{4.6^2 + 7^2} = \sqrt{70.16}
\]
\[
CD = \sqrt{(-8-8)^2 + (2-(-4))^2} = 6
\]
\[
DE = \sqrt{(-8-1.2)^2 + (-4-4)^2} = 6.8
\]
\[
EF = \sqrt{(-1.2-5)^2 + (-4-0)^2} = \sqrt{(-6.2)^2 + (-4)^2} = \sqrt{54.44}
\]
\[
FA = \sqrt{(5-8)^2 + (0-9)^2} = \sqrt{(-3)^2 + (-9)^2} = \sqrt{90}
\]

Perimeter = \(11.4 + \sqrt{70.16} + 6 + 6.8 + \sqrt{54.44} + \sqrt{90} \approx 49.44\)
Find the exact area of the two sectors.

Area of sector = \( \frac{\theta \cdot \pi \cdot (r)^2}{360} \)

\[
\begin{align*}
\text{Area of sector} &= \frac{278}{360} \cdot \pi \cdot (6)^2 \\
&= \frac{278}{360} \cdot 36 \pi \\
&= \frac{139}{5} \pi \text{ in}^2 \\
&= 27.8\pi \text{ in}^2
\end{align*}
\]
Find the length of the minor and major arcs.

\[ \text{length of arc} = \frac{\angle}{360} \cdot 2\pi r \]

\[ \text{length of arc} = \frac{82}{360} \cdot 2\pi \cdot 6 \]

\[ = \frac{41}{15} \pi \text{ in.} \]

\[ \text{length of arc} = \frac{278}{360} \cdot 2\pi \theta \]

\[ = \frac{139}{15} \pi \text{ in.} \]
Square ABCD is inscribed in circle P and has a side length of 12 meters. Find the area of the shaded region.

Area of circle = \( \pi r^2 \)
\[ = \pi \left( \sqrt{72} \right)^2 \]
\[ = 72\pi \text{ m}^2 \]

Area of square = \( l \cdot w \)
\[ = 12 \cdot 12 \]
\[ = 144 \text{ m}^2 \]

Area of shaded region = Circle - Square
\[ 72\pi - 144 \text{ m}^2 \]

\[ 12^2 + 12^2 = (2r)^2 \]
\[ 144 + 144 = 4r^2 \]
\[ 72 = r^2 \]
\[ \sqrt{72} = r \]