6.3 Sampling Distributions and CLT

Recall...

**Statistics**

- $\bar{X} = \text{Sample mean}$
- $S_x = \text{Sample standard deviation}$
- $n = \text{Sample size}$
  
  (# of data values in sample)

**Parameters**

- $\mu = \mu_x = \text{population mean}$
- $\sigma = \sigma_x = \text{Population Std deviation}$
- $N = \text{population size}$
Sampling Distribution:

a data display that shows the values of a certain statistic \( \frac{\text{like}}{X} \)

\[ \mu_X = \text{the mean of the sample means} \]
\[ u_X = u_X \]

\[ \sigma_X = \text{the standard deviation of the sample means} \]
\[ \sigma_X = \frac{\sigma_X}{\sqrt{n}} \quad \text{if } n \leq 10\% \text{ of } N \]
The sampling distribution of $\bar{x}$
is approx Normal if...

- The population distribution of $x$ is approx Normal

OR - $n \geq 30$ (according to the Central Limit Theorem)
Central Limit Theorem:

As $n$ increases, the sampling distribution for $\bar{X}$ will become normal (and 'skinnier')

* $\mu_{\bar{X}}$ stays the same ... $\mu_{\bar{X}} = \mu_X$

* $\sigma_{\bar{X}}$ gets smaller ... $\sigma_{\bar{X}} = \frac{\sigma_X}{\sqrt{n}}$
Ex.

Neilsen reported that children between the ages of 2 and 5 watch an average of 25 hours of television per week. Assume the variable is normally distributed and the standard deviation is 3 hours. If 20 children between the ages of 2 and 5 are randomly selected, find the probability that the mean of the number of hours they watch television will be greater than 26.3 hours.
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\[ \mu_x = 25 \quad \mu_{\bar{x}} = 25 \]
\[ \sigma_x = 3 \quad \sigma_{\bar{x}} = \frac{3}{\sqrt{20}} \]
\[ n = 20 \]

\[ \text{normalcdf}(26.3, 1000, 25, \frac{3}{\sqrt{20}}) \]

\[ \approx 0.0267 \]
Ex.

The average age of a vehicle registered in the United States is 8 years (or 96 months). Assume the standard deviation is 16 months. If a random sample of 36 vehicles is selected, find the probability that the mean of their age is between 90 and 100 months.
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\[
\begin{align*}
P(90 < \bar{x} < 100) &= \text{normalcdf}(90, 100, 96, \frac{16}{\sqrt{36}}) \\
&\approx 92\%
\end{align*}
\]

\[
\begin{align*}
\mu_x &= 96 \\
\sigma_x &= 16 \\
\sigma_{\bar{x}} &= \frac{16}{\sqrt{36}} \\
n &= 36
\end{align*}
\]