Practice with Point Estimators and Confidence Intervals

1. In each of the following settings, determine the point estimator you would use and calculate the value of the point estimate.
   a. The makers of a new golf ball want to estimate the median distance the new balls will travel when hit by a mechanical driver. They select a random sample of 10 balls and measure the distance each ball travels after being hit by the mechanical driver. Here are the distances (in yards):

      285 286 284 285 282 284 287 290 288 285

      $\text{median} = 285$  

      Use the sample median to estimate the population median

   b. The golf ball manufacturer would also like to investigate the variability of the distance travelled by the golf balls by estimating the interquartile range.

      IQR = 287 - 284 = 3  

      Use the sample IQR to estimate the population IQR

   c. The math department wants to know what proportion of its students own a graphing calculator, so they take a random sample of 100 students and find that 28 own a graphing calculator.

      $\hat{p} = \frac{28}{100} = 0.28$  

      Use the sample proportion to estimate the population proportion

2. Mrs. Quinn's class took an SRS of 102 pennies and discovered that 57 of the pennies were more than 10 years old.
   a. Calculate and interpret a 99% confidence interval for $p =$ the true proportion of pennies that are more than 10 years old.

      Use a one-sample $z$ interval for $p$  

      SRS  

      $n = 102 \leq 10\%$ of all pennies  

      $n\hat{p} \geq 10$ and $n\hat{p} \geq 10$  

      $57 \geq 10$ and $45 \geq 10$  

      $n=57$, $\hat{p} = \frac{57}{102} = 0.56$  

      $\hat{p} = \frac{57}{102} = 0.56$  

      $\hat{p} = \frac{45}{102} = 0.44$  

      $CL = 99\%$  

      $Z^* = 2.576$  

      CI: (0.432, 0.685)  

      We are 99% confident the true proportion of pennies that are more than 10 years old is between 0.432 and 0.685

   b. Is it plausible that exactly 60% of all the pennies in the collection are more than 10 years old? Explain.

      Yes, since 60% falls between the 99% confidence interval, it is a reasonable (plausible) value for $p$, the proportion of pennies that are more than 10 years old.
3. A large company is concerned that many of its employees are in poor physical condition, which can result in decreased productivity. To determine how many steps each employee takes per day, on average, the company provides a pedometer to 50 randomly selected employees to use for one 24-hour period. After collecting the data, the company statistician reports a 95% confidence interval of 4547 steps to 8473 steps.

a. Interpret the confidence interval.

The statistician is 95% confident that the true mean steps taken per employee per day is in the interval 4547 and 8473 steps.

b. What is the point estimate that was used to create the interval? What is the margin of error?

\[
\text{point estimate } \bar{x} = \frac{8473 + 4547}{2} = 6510 \text{ steps}
\]

\[
\text{margin of error } = \frac{8473 - 4547}{2} = 1963 \text{ steps}
\]

4. Use Table A (in your formula packet) to find the critical value \( z^* \) for a 96% confidence interval. Assume that the Large Counts condition is met.

\[ z^* = 2.054 \]

5. What critical value \( t^* \) should be used in constructing a confidence interval in the following settings?

a. A 90% confidence interval based on an SRS of size 10.

\[ t^* = 1.833 \]

b. A 99% confidence interval based on a random sample of 95 observations.

\[ t^* = 2.629 \]

6. Which of the following is a criterion for choosing a \( t \)-test rather than a \( z \)-test when making an inference about the mean of a population?

A. The standard deviation of the population is unknown.

B. The mean of the population is unknown. (of course \( \mu \) is unknown... that is what we are looking for)

C. The sample may not have been a simple random sample. (Criterion for both tests)

D. The population is not normally distributed. (could have \( n \geq 30 \)

E. The sample size is less than 100. (as long as \( n \leq 10\% N \)
7. Determine whether we can safely use a $t^*$ critical value to calculate a confidence interval for the population mean in each of the following settings.

a. We want to estimate the average time (in minutes) to order and receive a regular coffee at a local coffee shop. The time for five randomly selected visits to a local coffee shop are shown in the dotplot below.

b. We want to estimate the average height of high school students. The boxplot below shows the distribution of heights (in cm) for 100 randomly selected students.

c. A middle-school counselor wants to estimate how many minutes, on average, students spend doing homework. A histogram of the amount of time doing homework the previous evening for a random sample of 20 students is shown below.

8. A counselor wants to estimate the average number of text messages sent by students at his school during school hours. He wants to estimate $\mu$ at the 99% confidence level with a margin of error of at most 2 texts. A pilot study indicated that the number of texts sent during school hours has a standard deviation of about 9 texts. How many students need to be surveyed to estimate the mean number of texts sent during school hours with 99% confidence and a margin of error of at most 2 texts?

$$CL: 99\%$$

$$\text{margin of error} = 2$$

$$\bar{x} = 9$$

$$z^* = 2.576$$

$$n \geq 134.374$$

They should survey at least 135 students.
9. For their second semester project in AP® Statistics, Ann and Tori wanted to estimate the average weight of an Oreo cookie to determine if the average weight was less than advertised. They selected a random sample of 36 cookies and found the weight of each cookie (in grams). The mean weight was \( \bar{x} = 11.3921 \) grams with a standard deviation of \( s_x = 0.0817 \) grams.

a. Construct and interpret a 95% confidence interval for the mean weight of an Oreo cookie.

\[ n = 36 \]
\[ S_x = 0.0817 \]
\[ \text{Confidence Interval (CI): } \bar{x} \pm t^* \left( \frac{s}{\sqrt{n}} \right) \]
\[ 11.3921 \pm 2.030 \left( \frac{0.0817}{\sqrt{36}} \right) \]
\[ (10.705, 12.079) \]
We are 95% confident that the true average weight of an Oreo cookie is between 10.705 gms and 12.079 gms.

b. On the packaging, the stated serving size is 3 cookies (34 grams). Does the interval in part (a) provide convincing evidence that the average weight of an Oreo cookie is less than advertised? Explain.

\[ \frac{34 \text{ gms}}{3 \text{ cookies}} = 11.33 \text{ gms/Cookie} \]

Since this value is within the 99% confidence interval, the advertised value is plausible and we do not have convincing evidence that the average weight of an Oreo is less than advertised.

10. As part of their final project in AP® Statistics, Christina and Rachel randomly selected 18 rolls of a generic brand of toilet paper to measure how well this brand could absorb water. To do this, they poured 1/4 cup of water onto a hard surface and counted how many squares of toilet paper it took to completely absorb the water. Here are the results from their 18 rolls:

\[ \begin{array}{cccccccc}
29 & 24 & 27 & 28 & 21 & 25 & 26 & 22 & 23 \\
\end{array} \]

Construct and interpret a 99% confidence interval for \( \mu \) = the true mean number of squares of generic toilet paper needed to absorb 1/4 cup of water.

\[ n = 18 \]
\[ \bar{x} = 24.94 \]
\[ S_x = 2.859 \]
\[ \text{Confidence Interval (CI): } \bar{x} \pm t^* \left( \frac{s}{\sqrt{n}} \right) \]
\[ 24.94 \pm 2.098 \left( \frac{2.859}{\sqrt{18}} \right) \]
\[ (22.991, 26.897) \]
We are 99% confident that the true mean number of squares of generic toilet paper needed to absorb 1/4 cup of water is between 22.991 and 26.897 squares.