Inductive Reasoning, Conjecture and Counterexamples

Inductive Reasoning
- Making an educated guess on the next item in the pattern assuming that the pattern continues.

Conjecture
- The educated guess, or a statement, can be true or false.

Counterexample
- Example proving that the conjecture is not true.

Number Sequences – Make a conjecture about the next two numbers in the sequence.

Ex. \(1, 3, 5, 7, 9, 11, \frac{13}{\sqrt{z} + \frac{13}{\sqrt{z}}}, \frac{15}{\frac{15}{\sqrt{z}}}\), ......

Arithmetic Sequence
- Keep adding or subtracting the same number.

Ex. \(3, 6, 12, \frac{24}{\sqrt{2} \times \sqrt{2}}, \frac{48}{\sqrt{2} \times \sqrt{2}}, \frac{96}{\sqrt{2} \times \sqrt{2}}, \) ...... Geometric Sequence
- Keep multiplying or dividing by the same number.

Ex. \(4, 9, 16, 25, 36, \frac{49}{\sqrt{2} \times \sqrt{2}}, \frac{64}{\sqrt{2} \times \sqrt{2}}\), ...... Quadratic Sequence
- The number you keep adding goes up by 2.

Ex. \(1, 1, 2, 3, 5, 8, 12, \frac{20}{\sqrt{2} \times \sqrt{2}}, \frac{32}{\sqrt{2} \times \sqrt{2}}, \) ...... Fibonacci Sequence
- Add the previous two terms to get next term.

You Try!

Ex. \(7, 10, 13, 16, 19, \frac{22}{\sqrt{2} \times \sqrt{2}}, \frac{25}{\sqrt{2} \times \sqrt{2}}\), ...... Arithmetic

Ex. \(3, 6, 11, 18, 27, \frac{38}{\sqrt{2} \times \sqrt{2}}, \frac{51}{\sqrt{2} \times \sqrt{2}}\), ...... Quadratic
Shapes – Make a conjecture about the next shape in the sequence.

Ex.

\[
\begin{array}{cccc}
1 & 3 & 6 & 10 \\
+2 & +3 & +4 & +5 \\
& & & +6 \\
& & & 21 \\
& & & 7 \\
& & & 28
\end{array}
\]

Ex.

\[
\begin{align*}
\triangle & \quad \square & \quad \triangle & \quad \text{triangle}
\end{align*}
\]

State whether each conjecture is true or false. If false, give a counterexample.

Ex. All fractions have a value between 0 and 1. \( \frac{7}{2} \) is greater than 1.

Ex. The product of two integers is always positive. \(-3 \cdot 2 = -6\)

Ex. All prime numbers are odd numbers. The product of two whole numbers is a whole number. True.

You try!

[Diagram with shapes and numbers]