The 7 Basic Postulates of (Euclidean) Geometry:
Through Words and Illustration

1. Through any two points there is exactly one line(s).
   Draw two points – how many different lines can you draw that contain both points?

2. Through any three noncollinear points, there is exactly one plane(s).
   Draw three noncollinear points – how many different planes can you draw that contain all three points?

3. A line contains at least two point(s).
   Think – what is the least number of points you need in order to name a line?

4. A plane contains at least three noncollinear point(s).
   Think – what is the least number of noncollinear points you need in order to name a plane?

5. If two points lie in a plane, then the entire line containing those points lies on that plane.
   Draw a plane and two points in the plane. Then, draw the entire line that contains both points.

6. If two lines intersect, then their intersection is exactly one point.
   Draw two lines intersecting. Then describe their intersection.

7. If two planes intersect, then their intersection is a line.
   Draw two planes intersecting. Then describe their intersection. (Remember the activity with the intersecting index cards?)
Use the postulates to determine whether each statement is *always*, *sometimes*, or *never* true. **Cite the postulate** you used to determine your answer (which means ‘write the postulate word for word’).

- If the statement is *always true*, draw one illustration.
- If the statement is *sometimes true*, draw one illustration in which the statement is true, and one illustration in which the statement is not true.
- If the statement is *never true*, explain why it is never true.

1. **A line contains exactly one point.**  *never, you need 2 points to define a line.*

2. **Noncollinear points** $R$, $S$, and $T$ are contained in exactly one plane. *Always True*

   2. Through any three noncollinear points, there is exactly one plane.

3. **Any two lines** $j$ and $k$ intersect. [Hint: Try to write this as an if-then statement first.]

   *Sometimes true*

4. **If points** $G$ and $H$ are contained in plane $M$, then $\overline{GH}$ is perpendicular to plane $M$. *Never true; there are in the plane and therefore cannot be perpendicular to the plane.*

5. **Planes** $R$ and $S$ intersect at point $T$.

   *Never; two planes intersect and make a line, not a point.*

6. **If points** $A$, $B$, and $C$ are noncollinear, then segments $\overline{AB}$, $\overline{BC}$, and $\overline{CA}$ are contained in exactly one plane.

   *Always True; 2. Through any three noncollinear points, there is exactly one plane*
7. Three points determine a plane. 
   Sometimes true.

8. Points S, T, and U determine three lines.
   Sometimes:

In the figure at the right, \( \overrightarrow{AC} \) and \( \overrightarrow{BD} \) lie in plane \( j \), and \( \overrightarrow{BY} \) and \( \overrightarrow{CX} \) lie in plane \( k \). State the postulate that can be used to show each statement is true.

1. \( C \) and \( D \) are collinear
   5. If two points lie in a plane, then the entire line containing those points lies on that plane.

2. \( \overrightarrow{XB} \) lies in plane \( k \)
   5. If two points lie in a plane, then the entire line containing those points lies on that plane.

3. Points \( A, C, \) and \( X \) are coplanar.
   2. Through any three non-collinear points, there is exactly one plane.

4. \( \overrightarrow{AD} \) lies in Plane \( j \)
   5. If two points lie in a plane, then the entire line containing those points lies on that plane.
Postulate 1 says "Through any two points, there is exactly one line."

We might then conclude that **through any two points, there is exactly one line segment that connects them.**

Determine the number of segments that can be drawn connecting each pair of points. Which means...every point needs to be connected to every other point.

1. 

2. 4
   \[4 + 3 + 2 + 1 = 10\]

3. 
   \[8 + 7 + 6 + 5 + 4 + 3 + 2 + 1 = 36\]

4. Sixty dancers will dance at the opening ceremony of the next Olympics. The dancers, each connected to each of the other students with wide colored ribbons, will move in a circular motion. How many ribbons are needed? Remember the Handshake problem we did on the first day of school?

\[
\frac{n(n-1)}{2} = \frac{60 \cdot 59}{2} = 1770 \text{ ribbons}
\]